16.451 Lecture 3: Proton Magnetic Moment (μ)

15 Sept. 2005

For the **electron**, which is a true **pointlike spin- ½ particle** (as far as we know), quantum electrodynamics predicts that the magnetic moment is almost exactly equal to the Bohr magneton:

Classical Calculation: Griffiths E&M Problem 5.42

$$\boxed{\mu_B = \frac{e\hbar}{2m_e} = 5.78\times 10^{-5} \quad eV/T}$$

$$\boxed{\Delta E = -\vec{\mu}\,.\,\vec{B}}$$

This is usually expressed in terms of a "spin g-factor":

$$\boxed{\mu = g\, s\, \mu_B}$$

See F&H sec. 6.6!

with **g = 2 and s = ½** for the electron spin.

The exact measured value is: $\boxed{\mu_e/\mu_B = 1.001\,159\,652\,187 \pm 0.000\,000\,000\,004}$

The tiny discrepancy is referred to as the electron's **anomalous magnetic moment** or alternatively as the value of **"(g-2)"**. It arises from the electron's interaction with virtual particles in the vacuum, and can be calculated from first principles in QED. Agreement between theory and experiment is staggeringly good – better than 10^{-10}

Bottom line: a pointlike spin- ½ particle should have a g factor very close to 2.

Now for the proton:

By analogy, a **pointlike proton** should have a magnetic moment given by the **nuclear magneton,** μ_N:

$$\mu_N = \frac{e\hbar}{2m_p} = 3.15\times10^{-8}\ eV/T$$

that is, we expect:

$$\boxed{\mu_p = g\, s\, \mu_N}$$

with **g = 2 and s = ½** for the proton spin.

The exact measured value is: $\mu_p/\mu_N = \underline{2.7928}47351 \pm 0.000000028$

(PDG 2005)

which implies a *g*-factor of about 5.586 --- a **huge discrepancy** with the prediction for a pointlike object...

Conclusion: the proton must have an **internal structure** that accounts for its magnetic moment discrepancy

(quark model prediction: $\underline{2.793}\ \mu_N$, based on 3 pointlike, spin – ½ constituents!)

Measurements: I Early Days

SEPTEMBER 15, 1937 PHYSICAL REVIEW VOLUME 52

The Magnetic Moment of the Proton

I. Estermann, O. C. Simpson and O. Stern
Research Laboratory of Molecular Physics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania

(Received July 9, 1937)

The magnetic moment of the proton was measured by the method of the magnetic deflection of molecular beams employing H_2 and HD. The result is $\mu_P = 2.46\mu_0 \pm 3$ percent.

THE magnetic moment of the proton was first measured by Estermann, Frisch and Stern in Hamburg in 1932–33.[1] These measurements gave the surprising result that the proton moment was not one but 2.5 nuclear magnetons with the limit of error of about 10 percent. We have repeated these measurements with the aim of obtaining as great an accuracy as possible. The knowledge of this numerical value is important for several reasons: It allows a check on any theory of the heavy elementary particles, because the theory must give just this numerical value; but it is, of course, also important for the theory of the nuclei and for the theory of the forces between elementary particles.

Stern – Gerlach effect:

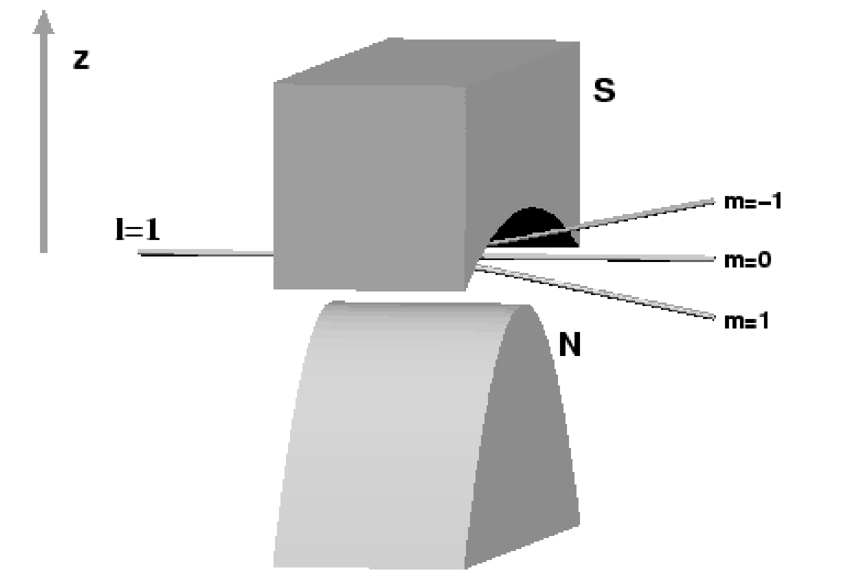
$$\vec{F} = -\vec{\nabla} U = \vec{\nabla}(\vec{\mu} \bullet \vec{B})$$

Deflection in an inhomogeneous magnetic field is proportional to the magnetic moment.

- H_2 molecule exists in two angular momentum states:

  Ortho-H_2 ($J = 1$) and **Para-H_2 ($J = 0$)**

- room temperature gas is $\frac{3}{4}$ ($J = 1$) and $\frac{1}{4}$ ($J = 0$) *(unpolarized, random orientations)*
- ($J = 1$) component has proton spins parallel and electron spins coupled to zero
- **magnetic moment points along the spin direction** for protons: **μ(ortho-H_2) = 2 μ_p**
- beam should separate into 3 separate components in a nonuniform B – field, corresponding to $m_J = (1, 0, -1)$ with separations proportional to μ_p



Deflections:

Let B_z be the main magnetic field component. The force on each magnetic substate is:

$$F_z = \frac{\partial}{\partial z}\left\langle \vec{\mu} \bullet \vec{B} \right\rangle = \left\langle \mu_z \right\rangle \frac{\partial B_z}{\partial z} = 2 g_p \, m_J \, \frac{\partial B_z}{\partial z}$$

→ m = 0 substate is not deflected; the m = + 1 and -1 substates are deflected equal amounts in opposite directions.

Some details ...

- had to account for magnetic moment associated with molecular rotation – complications reduced by limiting rotational excitations at low temperature (90 K)
- temperature, pressure and geometry dependence of the lineshape carefully modelled
- results relied on **absolute magnetic field map**

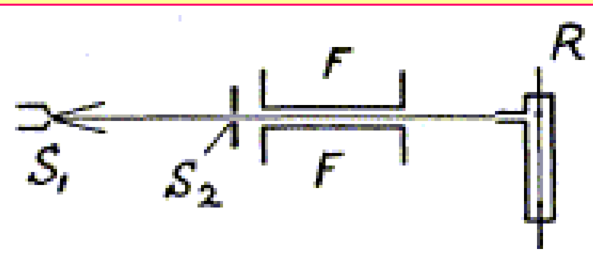


FIG. 7. Schematic diagram of the apparatus. S_1, source slit. S_2, collimating slit. F, magnetic field. R, receiver.

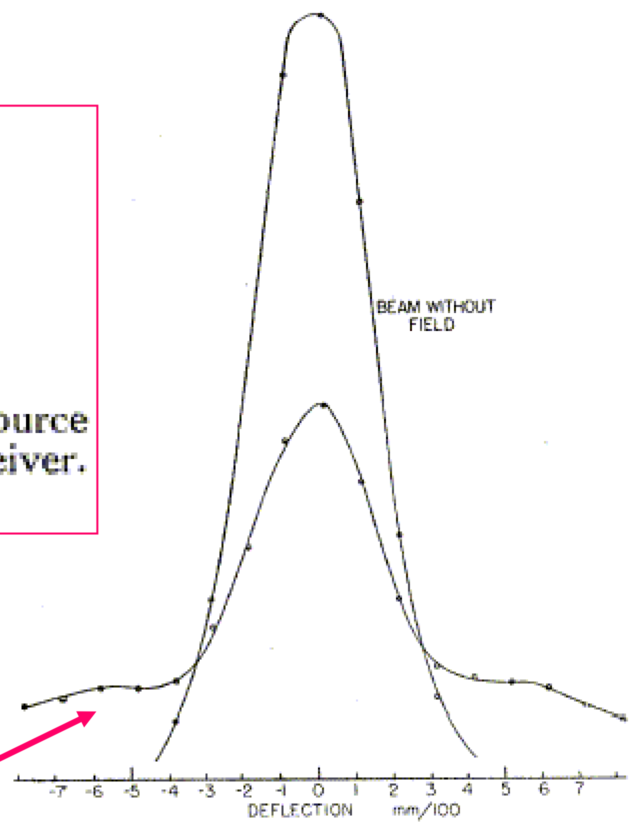


FIG. 3. Magnetic deflection of a beam of H_2 ($T=90°$K). Intensity in arbitrary units.

Result: $\mu_p = 2.46 \pm 0.07\ \mu_N$

not a huge signal!

Modern measurement, II: spectroscopy of atomic hydrogen

(Clever trick allows for a comparison of states without knowing the B field exactly!)

- **hydrogen atom ground state** has a hyperfine structure due to interaction of the electron and nuclear spins
- total angular momentum of the atom: $\boxed{\vec{F} = \vec{s}_e + \vec{s}_p}$

Addition of angular momentum:

$$m_F = m_e + m_p$$

state	m_e	m_p	m_F
1	$\frac{1}{2}$	$\frac{1}{2}$	1
2	$\frac{1}{2}$	$-\frac{1}{2}$	0
3	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
4	$-\frac{1}{2}$	$\frac{1}{2}$	0

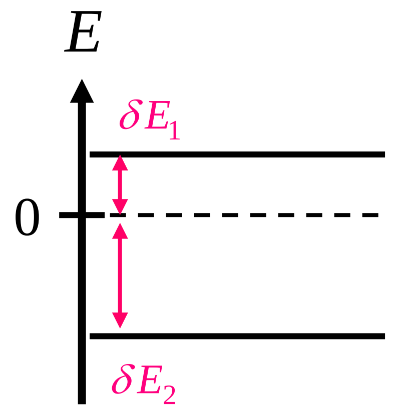
Total angular momentum has eigenvalues F = 0, 1. F=1 has 3 magnetic substates (1, 0, -1) and F = 0 has only 1 (m_F = 0)

(Counting up the complete set of Configurations always allows one to deduce the set of final state quantum numbers.)

Hyperfine splitting of hydrogen atomic states in zero external field:

origin of the effect: magnetic field of electron couples to magnetic moment of the nucleus (proton), affecting the total energy via $\delta E = -\vec{\mu} \bullet \vec{B}$

Energy of the hydrogen atom in zero magnetic field.



$F = 1, \; m_F = (1, 0, -1)$

$F = 0, \; m_F = 0$

$$\boxed{\delta E_F = \alpha \left\langle \vec{s}_e \bullet \vec{s}_p \right\rangle_F}$$

$$\boxed{\delta E_1 + \delta E_2 = 5.8 \times 10^{-6} \; eV}$$

Understandable theory:
D.J. Griffiths, Am. J. Phys. 50(8), 1982

NB. F = 1 to F = 0 transitions correspond to the famous "21 cm line", one of the most prevalent transition radiations in the universe!

Atomic Hydrogen energy levels split in an **external** magnetic field:

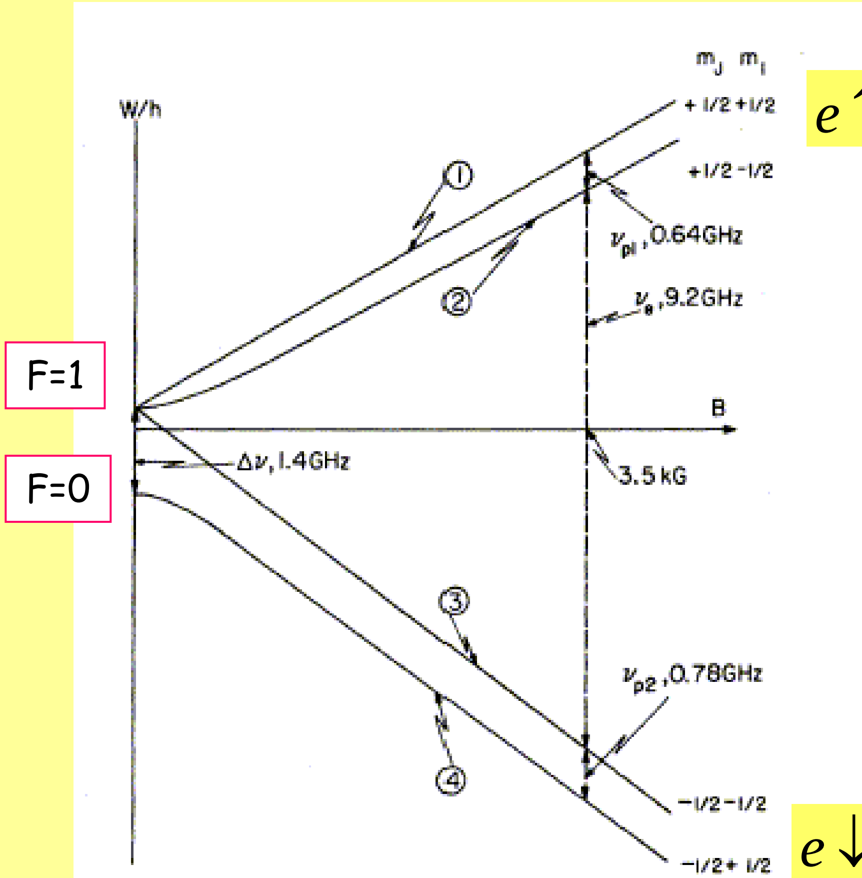


FIG. 1. Energy levels of hydrogen in the ground electronic state in an applied magnetic field.

$e\uparrow$

$p\uparrow,\ m_F = 1 \ \therefore\ F = 1$

$p\downarrow,\ m_F = 0$

The electron's magnetic moment is almost 2000x larger than that of the proton and of opposite sign. The main splitting between states at large B is due to the electron.

The hyperfine interaction between the electron and proton spins still causes the F = 1 state to have higher energy. This explains the order of states in the diagram.

$e\downarrow$

$p\downarrow,\ m_F = -1 \ \therefore\ F = 1$

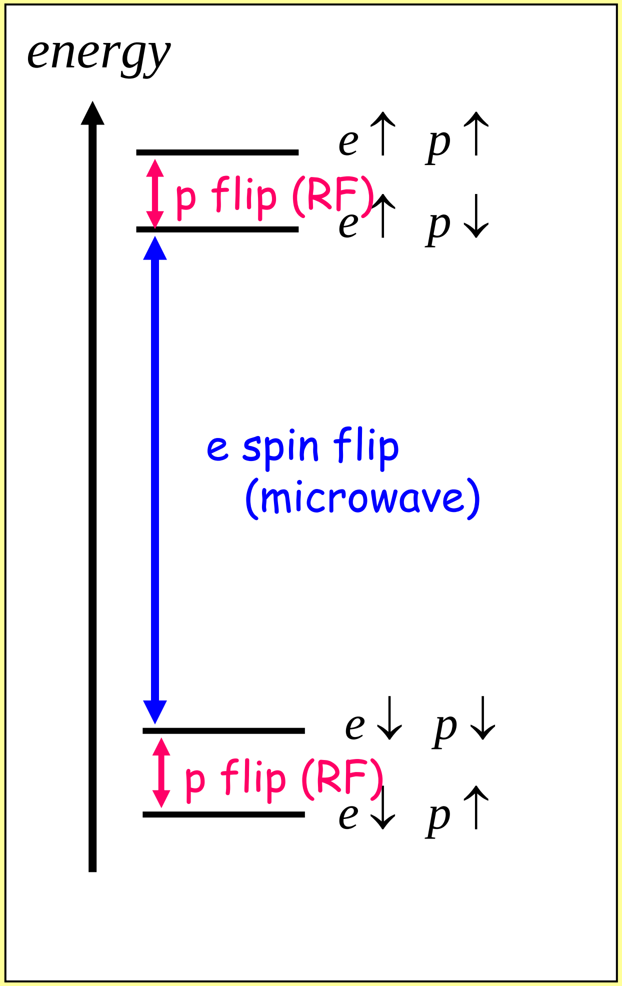
$p\uparrow,\ m_F = 0$

Idea for a precision measurement:

• select the states with particular electron spin direction by passing an atomic beam through a nonuniform magnetic field (Stern- Gerlach effect) (eg lower states in the diagram)

• irradiate the atoms with EM radiation at two frequencies simultaneously and determine the conditions for simultaneous absorption at both frequencies, i.e. flip both the electron and the proton spins. If the bandwidth of the two imposed EM sources is sufficiently narrow, an atom can ONLY absorb the microwave radiation if it simultaneously absorbs the RF

• the RATIO of electron and proton magnetic moments is obtained to high precision from the ratio of the two transition frequencies, without having to know the value of B.



PHYSICAL REVIEW A VOLUME 5, NUMBER 1 JANUARY 1972

Magnetic Moment of the Proton in Bohr Magnetons*

P. Frank Winkler,† Daniel Kleppner,‡ Than Myint,§ and Frederick G. Walther‡
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 8 September 1971)

The magnetic moment of the proton has been measured by observing simultaneously an electronic and a nuclear magnetic transition in atomic hydrogen. Observations were made with a hydrogen maser operating in a 3500-G field. A theory is presented for the transient response of a three-level system under conditions of double resonance including effects of cavity pulling, spin-exchange collisions, and frequency shifts due to motional field narrowing. The electron-proton g-factor ratio in hydrogen is found to be $g_J(\mathrm{H})/g_p(\mathrm{H}) = \mu_J(\mathrm{H})/\mu_p(\mathrm{H}) = -658.210\,706\,(6)$. This leads to a value of the proton moment in Bohr magnetons of $\mu_p/\mu_B = 1.521\,032\,181\,(15) \times 10^{-3}$.

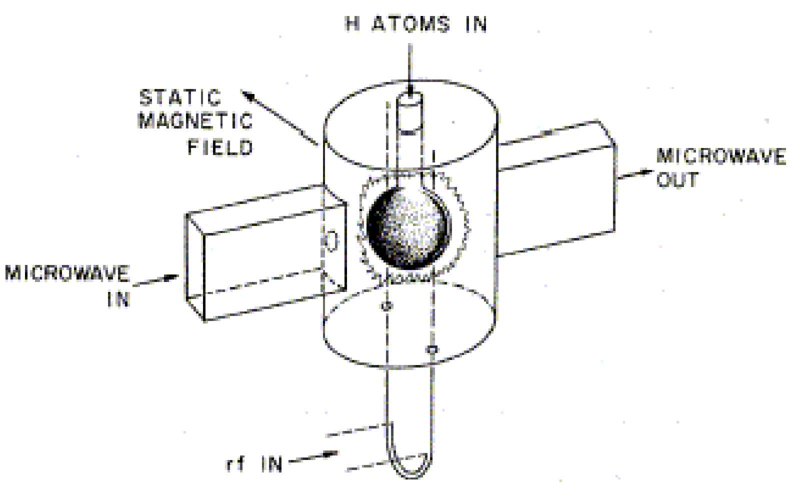


FIG. 11. Cavity and storage bulb.

Double resonance – microwaves show absorption dip when RF frequency is just right to flip proton spins. States are "filtered" beforehand so that only the double resonance can lead to a dip in the microwave signal.

Result: μ_p/μ_e measured to a relative accuracy of 10^{-8}!

Note - A hydrogen maser is required to flip the electron spins – Ramsey, Nobel prize 1989, shared with the "trappers" discussed in lecture 2.

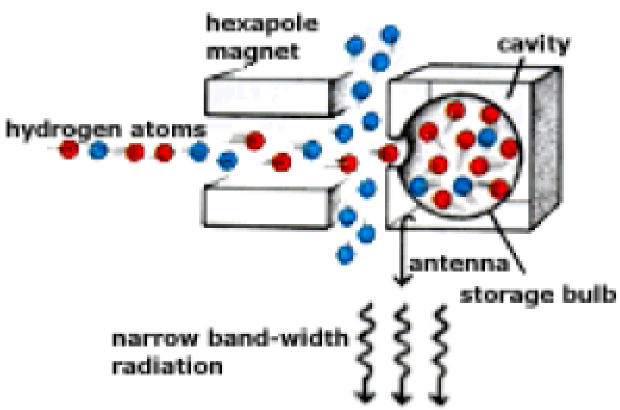


Fig. 12

The **hydrogen maser** (Ramsey) is another atomic clock (fig. 12). In this case the excited hydrogen atoms are selected by a hexapole magnet (Paul). These atoms are directed into a cavity that is part of an electric circuit tuned to the same resonant frequency as the radiation emitted by the excited hydrogen atoms. The radiation energy built up by the atoms causes the cavity to oscillate. The cavity can be connected to an antenna, the signal of which has a frequency stability of 1×10^{-15}. When measuring continental drifts or checking Einstein's general relativity theory, it is in fact more important to have a clock with high stability than to know its exact frequency.

Spin state selection via Stern -Gerlach effect (hexapole magnet) Used to prepare states for the Magnetic moment measurement:

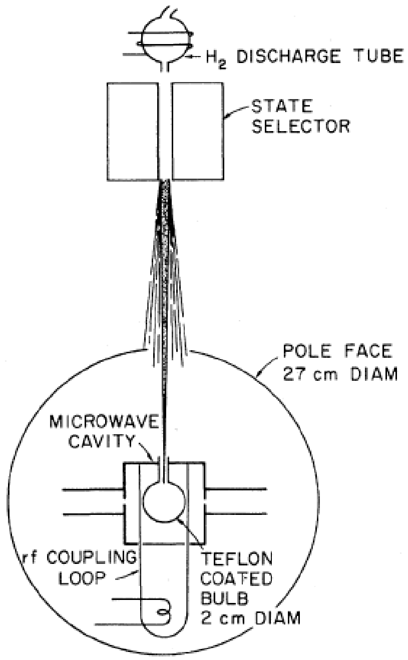


FIG. 9. Schematic diagram of the apparatus.

Currently accepted value of the proton magnetic moment (2005):

REVIEWS OF MODERN PHYSICS, VOLUME 77, JANUARY 2005

CODATA recommended values of the fundamental physical constants: 2002*

Peter J. Mohr[†] and Barry N. Taylor[‡]

National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8401, USA

(Published 18 March 2005)

This paper gives the 2002 self-consistent set of values of the basic constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology (CODATA) for international use. Further, it describes in detail the adjustment of the values of the subset of constants on which the complete 2002 set of recommended values is based. Two noteworthy additions in the 2002 adjustment are recommended values for the bound-state rms charge radii of the proton and deuteron and tests of the exactness of the Josephson and quantum-Hall-effect relations $K_J=2e/h$ and $R_K=h/e^2$, where K_J and R_K are the Josephson and von Klitzing constants, respectively, e is the elementary charge, and h is the Planck constant. The 2002 set replaces the previously recommended 1998 CODATA set. The 2002 adjustment takes into account the data considered in the 1998 adjustment as well as the data that became available between 31 December 1998, the closing date of that adjustment, and 31 December 2002, the closing date of the new adjustment. The differences between the 2002 and 1998 recommended values compared to the uncertainties of the latter are generally not unreasonable. The new CODATA set of recommended values may also be found on the World Wide Web at physics.nist.gov/constants.

Referenced by the Particle Data Group (2005)

a. Electron to proton magnetic moment ratio μ_e/μ_p

The ratio μ_e/μ_p is obtained from measurements of the ratio of the magnetic moment of the electron to the magnetic moment of the proton in the 1S state of hydrogen $\mu_{e^-}(\mathrm{H})/\mu_p(\mathrm{H})$. We use the value obtained by Winkler *et al.* (1972) at MIT:

$$\frac{\mu_{e^-}(\mathrm{H})}{\mu_p(\mathrm{H})} = -658.210\,7058(66) \quad [1.0\times 10^{-8}], \tag{56}$$

where a minor typographical error in the original publication has been corrected (Kleppner, 1997). The free-particle ratio μ_e/μ_p follows from the bound-particle ratio and the relation

$$\frac{\mu_{e^-}}{\mu_p} = \frac{g_p(\mathrm{H})}{g_p}\left(\frac{g_{e^-}(\mathrm{H})}{g_{e^-}}\right)^{-1}\frac{\mu_{e^-}(\mathrm{H})}{\mu_p(\mathrm{H})}$$
$$= -658.210\,6860(66) \quad [1.0\times 10^{-8}], \tag{57}$$

where the bound-state g-factor ratios (and all others needed in this section) are given in Table XLI in Appendix D. The stated standard uncertainty is due entirely to the uncertainty of the experimental value of $\mu_{e^-}(\mathrm{H})/\mu_p(\mathrm{H})$, because the bound-state corrections are taken as exact.

Taken from the 1972 paper by Winkler et al, as studied here!